## Model Examination paper-I

1(i) Let $(\omega, \mathcal{F}, P)$ be a probability space. Prove or disprove the following. If $A, B \in \mathcal{F}$ and $P(AB) = 0$, then $AB = emptyset$. [5]

1(ii) Define a filed and give an example of a field which is not a $\sigma$-field. [5]

2(i) Let $X$ be a random variable. Is $X$ is independent of $|X|$ iff $X$ is a constant random variable. [5]

2(ii) Check whether the following function is a distribution function.

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x & \text{if} \quad 0 \leq x \leq \frac{1}{2} \\ 1 & \text{if} \quad x > \frac{1}{2} \end{cases}$$

[5]

3(i) Let $X$ be a normal random variable with mean $1$ and variance $2$. Find $a, b \in \mathbb{R}$ such that $a + bX$ is standard normal. [5]

3(ii) Let $X$ and $Y$ be independent and identically distributed geometric random variables with parameter $p$. Find $P\{X \geq Y\}$ and $p\{X = Y\}$. [5]

4(i) Let $X$ be a Poisson random variable with mean $\lambda$. Show that

$$P\{X \leq \frac{\lambda}{2}\} \leq \frac{4}{lambda}.$$

[5]

4(ii) Let $X, Y$ be continuous random variable with joint pdfs

$$f(x, y) = \begin{cases} 4e^{-2y}, \ 0 \leq x \leq y \\ 0 \text{ otherwise} \end{cases}$$

Find the marginal pdfs of $X and\ Y$. [5]

5(i) Let $X_n, n = 1, 2, \ldots$ be a sequence of random variables. Show that $X_n \to 0$ in distribution iff $X_n \to 0$ in probability. [7]

5(ii) State central limit theorem. [3]